

Scheduling of multiple tandem queues in broadcasting and cable TV sector

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Abstract

The broadcasting and cable TV services sector has exhibited consistent growth over years spanning the last two decades. The sector comprises of analogue and digital cable TV services, DTH services, terrestrial TV services, IPTV services, and Radio services. The FM radio services has also demonstrated consistent growth. Commensurate to a growth in the subscriber base, the number of service providers has also increased. The development status of various services in the broadcasting sector is outlined below.

Keywords: tandem model, DTH, IPTV, FM

Introduction

Consider a system with a dynamic audience interested in a common broadcast from a central server. This can be modeled as a queuing system in continuous time with customers (the audience) arriving according to a Poisson process of intensity λ . The server has the ability to service the audience with broadcasts separated by an exponentially distributed random duration, whose maximum rate is μ (and can be controlled to any value between 0 and μ). Whenever a broadcast is made, the entire audience present in the system at that instance is served (i.e., the total number of customers is reduced to 0 at that instance). There are non-negative costs associated with each broadcast, and also for holding customers in the system, which is specified by a cost rate function that depends on the number of customers in the system at any given time. This cost rate function could be for instance, linear, or more generally even convex in the number of customers waiting. Our aim is to minimize the infinite horizon discounted cost, and to understand the structure of the associated optimal policies. The general broadcast scheduling problem arises in applications where a central server has multiple pages with customer requests for each page arriving independently. Each service can satisfy all outstanding requests for any single page. The aim is usually to minimize either the average waiting time for the page requests, or to minimize the maximum waiting time. A large body of work in the database and algorithms literature has focused on scheduling for the broadcasting model (eg: [6], [5], [7], [4], [8], [2]). A strong emphasis is on competitive analysis, in which oblivious online policies and optimal offline algorithms that have a precise knowledge of the future sample path of the customer arrivals are analyzed. To put the stochastic control model in perspective with this alternate line of investigation, one could view this as a middle ground between the omniscient offline algorithm and the completely oblivious online algorithm, since the assumption of Poisson arrivals amounts to a partial knowledge on the sample paths of customer arrivals. In this context, [1] studies the batch processing problem in multiple queues. A variant with constant service time was studied also in [30].

Scheduling of Multiple Tandem Queues

In this section we shall consider the case of two queues. Unlike the single queue case for which we were able to tackle the convex cost model (and the monotone cost model was handled), we will only consider a monotone cost in the number of waiting customers for two queues. More explicitly, there are two classes of customers who arrive according to independent Poisson processes of rates λ_1 and λ_2 . We also have a broadcasting server of rate μ . Assume without loss of generality that $\lambda_1 + \lambda_2 + \mu = 1$. There is no service cost. Let the cost rate be given as $c(x_1; x_2)$ when the number of customers in queues 1 and 2 is $x_1; x_2$ respectively. We shall assume that c is non decreasing in $(x_1; x_2)$. Again, although a continuous time system, the time integrals of the instantaneous cost (both discounted as well as long run average) can be conveniently cast in terms of the discrete time jump processes because of the independence of inter-event times with respect to the states (which comes from the Poisson arrivals and service processes). Let β be the equivalent discount factor for this discrete time problem. Let c_1 and c_2 be the equivalent service costs for queue 1 and 2 respectively for the equivalent discrete time problem. The n step cost function starting at state $(x_1(0); x_2(0))$ is (where u notes control and the evolution of the state is implicitly as per control u):

$$V_n(x_1, x_2) = \inf_u E_{(x_1(0), x_2(0))}^u \sum_{k=0}^{n-1} \beta^k c(x_1(k), x_2(k))$$

Via dynamic programming, we can recursively characterize V_n as: $V_0=0$,

$$\text{and } V_{n+1}(x_1, x_2) = c(x_1, x_2) + \beta \{ \lambda_1 V_n(x_1+1, x_2) + \lambda_2 V_n(x_1, x_2+1) + \mu \min(V_n(x_1, 0), V_n(0, x_2)) \} \quad (6.1)$$

The optimal control action with n steps to go at state $(x_1; x_2)$ is given by:

$$u_n(x_1, x_2) = \begin{cases} 2, & \text{if } V_n(x_1, 0) \leq V_n(0, x_2) \\ 1, & \text{otherwise} \end{cases} \quad (6.2)$$

In the above description, the control variable $u_n(x_1, x_2)$ denotes the queue to be served at state (x_1, x_2) Remark 1: By letting $n \rightarrow \infty$ it can be argued that v_∞ exists and v_n converges to it, and v_1 also inherits the properties of v_n that are shown below via induction, including the switching structure, because the set of functions satisfying them is closed under point-wise limits.

$$\text{Let } (x_1, x_2) < (y_1, y_2) \Leftrightarrow x_1 \leq x_2 \quad \text{and} \quad y_1 \leq y_2$$

Lemma 6.1: For any

$$x \in Z_+^2, y \in Z_+^2 \quad \text{suchthat} \quad x < y, V_n(x) \leq V_n(y).$$

Proof: Let $x = (x_1, x_2), y = (y_1, y_2)$ be such that $x < y$. The assertion holds for $n=0$ from the monotonicity of c . We also have:

$$\begin{aligned} &V_{n+1}(y) - V_{n+1}(x) = c(y_1, y_2) - c(x_1, x_2) + \beta \lambda_1 (V_n(y_1 + 1, y_2) - V_n(x_1 + 1, x_2)) \\ &+ \beta \lambda_2 (V_n(y_1, y_2 + 1) - V_n(x_1, x_2 + 1)) \\ &+ \beta \mu (\min(V_1(y_1, 0), V_n(0, y_2)) - \min(V_1(x_1, 0), V_n(0, x_2))) \end{aligned}$$

If $x < y$, we also have

$$(x_1 + 1, x_2) < (y_1 + 1, y_2), (x_1, 0) < (y_1, 0), \text{etc.}$$

If the assertion holds for n , one can easily check that this implies that each of the above terms is non-negative. Therefore, it also holds for $n + 1$.

Theorem 6.2: The optimal control with n steps to go is given by a switch curve:

$$u_n(x_1, x_2) = \begin{cases} 2 & \text{if } x_2 \geq s_n(x_1) \\ 1 & \text{otherwise} \end{cases} \quad (6.3)$$

Where

$$s_n(x) = \min \{y : V_n(x, 0) \leq V_n(0, y)\} \quad (6.4)$$

Proof: Follows from interpreting equation (6.2) using Lemma 6.1.

Conclusion

The broadcast service tandem queuing model which corresponds to batch processing with a batch size infinity was considered. This increase in the subscriber base in all these years has been aptly addressed because of the Multiple queuing tandem. The overwhelming response of the subscribers has been only because of the matching service base. With a number of broadcasting companies like Tata Sky, Airtel, Dish TV, Videocon D2H, etc, the competition has been cut throat. With a number of options that are available to the customers in terms of monetary plans, in terms of quality, and in terms of quantity, the awareness levels have been rising every year. Consequently, the desirability for the broadcasting companies to provide better and cheaper plans has been increasing simultaneously. To answer all these demands in the

best possible manner is the first and foremost success formula for all of these companies. The need of the hour is to increase the subscriber base without compromising with the present subscriber base. For this, multiple queuing tandems have been posing as the suited saviors. For instance, had multiple queuing tandems not been there, this increasing demand would have been left un-supplied for.

Multiple queuing tandems help create various entries and escapes for serving all these increasing demands simultaneously, unlike the three-station and two-station queuing systems that make the increasing demands wait unless a service is free to be rendered owing to the scarce number of services available.

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